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Dark Energy in Global Brane Universe

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We discuss the exact solutions of brane universes and the results indicate the Friedmann equations on the branes are modified with a new density term. Then, we assume the new term as the density of dark energy. Using Wetterich's parametrization equation of state (EOS) of dark energy, we obtain the new term varies with the red-shift z . Finally, the evolutions of the mass density parameter Ω_2 , dark energy density parameter Ω_x and deceleration parameter q_2 are studied.

Keywords: dark energy, brane universe

1. Introduction

It is proposed that our universe is a 3-brane embedded in a higher-dimensional space.^{1–8} In this brane world model, gravity can freely propagate in all dimensions, while standard matter particles and forces are confined on the 3-brane. A five-dimensional (5D) cosmological model and derived Friedmann equations on the branes are considered by Binetruy, Deffayet and Langlois (BDL),⁹ for a recent review, it can be seen.^{10,11} Recent observations indicate that our universe is accelerating^{12,13} and dominated by a negative pressure component dubbed dark energy. Obviously, a natural candidate is the cosmological constant with equation of state $w_\Lambda = -1$. But, comic observations imply that dark energy may be dynamic.^{14–16} So, a lot of dark energy models are studied extensively, such as quintessence, phantom, etc.^{14–17} While, brane-world models of dark energy are studied¹⁸ and accelerating universe comes from gravity leaking to extra dimension.¹⁹ In this paper, we study the dark energy and the universe evolution on the branes which are embedded in a Ricci-flat bulk characterized by a class of exact solutions. The solutions were firstly presented by Liu and Mashhoon and restudied latter by Liu and Wesson.^{20,21} And they are algebraically rich because they contain two arbitrary functions of time t . Then they are studied as Ricci-flat universe widely^{22–29} and exact global solutions of brane universes.³⁰

In this paper, we discuss dark energy in global brane universes. The exact global solutions of brane universes are studied and they show that that the Friedmann

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equations on the second brane ($y \neq 0$) are modified with a new density term. Then we assume this term as the density of dark energy. Since the EOS of dark energy has been presented and investigated widely,^{31–38} now we use the Wetterich's parametrization EOS of dark energy³⁹ to study the dark energy on the brane $y \neq 0$ (the second brane). This paper is organized as follows: In Section II, we derive the Friedmann equations with a new term on the second brane from the exact global solutions and assume this new term as the density of dark energy. In Section III, the evolutions of the dimensionless density parameters of matter Ω_2 and dark energy Ω_x respectively and deceleration parameter q_2 on the branes $y \neq 0$ are obtained by using Wetterich's parametrization EOS of dark energy to study dark energy. Section IV is a short conclusion.

2. Friedman equations in brane universes

The 5D cosmological solutions^{20,21} read

$$dS^2 = B^2 dt^2 - A^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right) - dy^2, \quad (1)$$

where $d\Omega^2 = d\theta^2 + \sin^2 \theta d\psi^2$ and k is the 3D curvature index ($k = \pm 1, 0$). For the solutions satisfy the 5D vacuum equation $R_{AB} = 0$, they are used as the bulk solutions of BDL-type brane model. To obtain brane models for using the Z_2 reflection symmetry on A and B , they are set as³⁰

$$\begin{aligned} A^2 &= (\mu^2 + k) y^2 - 2\nu |y| + \frac{\nu^2 + K}{\mu^2 + k}, \\ B &= \frac{1}{\mu} \frac{\partial A}{\partial t} \equiv \frac{\dot{A}}{\mu}, \end{aligned} \quad (2)$$

where $\mu = \mu(t)$ and $\nu = \nu(t)$ are two arbitrary functions, and K is a constant.

Then the corresponding 5D bulk Einstein equations are taken as

$$\begin{aligned} G_{AB} &= \kappa_{(5)}^2 T_{AB}, \\ T_B^A &= \delta(y) \text{diag}(\rho_1, -p_1, -p_1, -p_1, 0) \\ &\quad + \delta(y - y_2) \text{diag}(\rho_2, -p_2, -p_2, -p_2, 0) \end{aligned} \quad (3)$$

where the first brane is at $y = y_1 = 0$ and the second is at $y = y_2 > 0$. In the bulk $T_{AB} = 0$ and $G_{AB} = 0$, Eq.(3) are satisfied by (2). On the branes Liu had solved Eq.(3) in Ref. 30. We adopt the result at $y = y_1 = 0$ and $y = y_2 > 0$ as follows:

$$\kappa_{(5)}^2 \rho_1 = \frac{6\nu}{A_1^2}, \quad (4)$$

$$\kappa_{(5)}^2 p_1 = -\frac{2}{\dot{A}_1} \frac{\partial}{\partial t} \left(\frac{\nu}{A_1} \right) - \frac{4\nu}{A_1^2}, \quad (5)$$

and

$$\kappa_{(5)}^2 \rho_2 = \frac{6}{A_2} \left(\frac{\mu^2 + k}{A_2} y_2 - \frac{\nu}{A_2} \right), \quad (6)$$

$$\begin{aligned} \kappa_{(5)}^2 p_2 = & -\frac{2}{\dot{A}_2} \frac{\partial}{\partial t} \left(\frac{\mu^2 + k}{A_2} y_2 - \frac{\nu}{A_2} \right) \\ & - \frac{4}{A_2} \left(\frac{\mu^2 + k}{A_2} y_2 - \frac{\nu}{A_2} \right). \end{aligned} \quad (7)$$

Now, we consider the universe on the second brane, i.e. $y = y_2 > 0$. From the 5D metric (1), the Hubble and deceleration parameters on $y = y_2$ brane can be defined as

$$H_2(t, y) \equiv \frac{1}{B_2} \frac{\dot{A}_2}{A_2} = \frac{\mu}{A_2}, \quad (8)$$

$$q_2(t, y) = -\frac{A_2 \dot{\mu}}{\mu \dot{A}_2}. \quad (9)$$

Substituting Eq.(8) into Eq.(6) to eliminate μ^2 , we find that Eq. (6) can be rewritten into a new form as

$$H_2^2 + \frac{k}{A_2^2} = \frac{\kappa_{(5)}^2}{6y_2} \left(\rho_2 + \frac{6}{\kappa_{(5)}^2} \frac{\nu}{A_2^2} \right). \quad (10)$$

Comparing with the standard Friedman equation i.e. $H^2 + \frac{k}{A^2} = \frac{\kappa_{(4)}^2}{3} \rho$, we can find $\kappa_{(4)}^2 = \kappa_{(5)}^2 / (2y_2)$. Since $\kappa_{(5)}^2 = M_{(5)}^{-3}$ and $\kappa_{(4)}^2 = M_{(4)}^{-2}$, the relation of four dimensional Planck mass is expressed with five dimensional Planck mass as

$$M_{(4)}^2 = 2y_2 M_{(5)}^3. \quad (11)$$

Therefore, the four dimensional Planck mass is relevant to five dimensional Planck mass and the position of brane. This Friedmann equation is different from BDL's because it is derived from the exact solution of 5D vacuum equation $R_{AB} = 0$ on $y \neq 0$ brane.

We assume the term $\frac{6}{\kappa_{(5)}^2} \frac{\nu}{A_2^2}$ as the density of dark energy. That is,

$$\rho_x = \frac{6}{\kappa_{(5)}^2} \frac{\nu}{A_2^2}. \quad (12)$$

From the Eq.(7), it is obtained

$$\frac{2\mu\dot{\mu}}{A_2\dot{A}_2} + \frac{\mu^2 + k}{A_2^2} = -\frac{\kappa_{(5)}^2}{2y_2} (p_2 + p_x), \quad (13)$$

where $p_x = -\frac{2}{\kappa_{(5)}^2} \left(\frac{\dot{\nu}}{A_2\dot{A}_2} + \frac{\nu}{A_2^2} \right)$. This is the pressure of dark energy. Then, from Eq. (9), Eq. (10) and Eq. (13), for $k = 0$, the deceleration parameters q_2 can be obtained

$$q_2 = \frac{1}{2} \left[\frac{3(p_2 + p_x)}{\rho_2 + \rho_x} + 1 \right]. \quad (14)$$

Meanwhile, the conservation law $T_{A;B}^B = 0$ gives

$$\dot{\rho}_2 + 3(\rho_2 + p_2) \frac{\dot{A}_2}{A_2} = 0. \quad (15)$$

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3. Density parameters and their evolution

From the definement of ρ_x and p_x , for $k = 0$, we obtain the EOS of dark energy

$$w_x = \frac{p_x}{\rho_x} = -\frac{1}{3}\left(\frac{A_2\dot{\nu}}{A_2\nu} + 1\right). \quad (16)$$

For a given component, which has the equation of state $p_2 = w_2\rho_2$ (with w_2 being a constant), we obtain $\rho_2 = \rho_{20}A_2^{-3(1+w_2)}$ from Eq.(15). Now, considering the given component as matter, i.e. $w_2 = 0$, we get $\rho_2 = \rho_{20}A_2^{-3}$. Therefore, the dimensionless density parameters are obtained

$$\Omega_2 = \frac{\rho_{20}}{\rho_{20} + \frac{6}{\kappa_{(5)}^2}\nu A_2}, \quad (17)$$

$$\Omega_x = 1 - \Omega_2, \quad (18)$$

here ρ_{20} is the current values of matter.

In terms of Eq.(2) A_2 is a function of t and y . However, on the given $y = y_2$ brane, A_2 becomes $A_2 = A_2(t)$. Furthermore, we use the relation

$$A_2 = \frac{A_{20}}{1+z}, \quad (19)$$

and define $\nu = \nu_0 f(z)$, and then we find that Eqs. (16)-(18) can be expressed via redshift z as

$$w_x = \frac{(1+z)}{3f(z)} \frac{df(z)}{dz} - \frac{1}{3}, \quad (20)$$

$$\Omega_2 = \frac{\Omega_{20}(1+z)}{\Omega_{20}(1+z) + (1-\Omega_{20})f(z)}, \quad (21)$$

$$\Omega_x = 1 - \Omega_2. \quad (22)$$

From Eqs. (20)-(22), the evolution of cosmic components will be determined if the function $f(z)$ is given.

Now we utilize the form of EOS of the dark energy given by Wetterich,³⁹ which has been studied.⁴⁰⁻⁴² The form is

$$w_x(z, b) = \frac{w_0}{1 + b \ln(1+z)}, \quad (23)$$

where $w_x(z, b)$ is the EOS parameter with its current value as w_0 , and b is a bending parameter describing the deviation of w_x from w_0 as z increases. By substituted Eq.(23) into Eq. (20), the function $f(z)$ is obtained as follows:

$$f(z) = (1+z)[1 + b \ln(1+z)]^{3w_0/b}. \quad (24)$$

Substituting Eq. (24) into Eq. (20), we can obtain (23). The only difference is $b \neq 0$ in this condition. So, there must be deviation from w_0 as redshift z . We plot the evolution of the EOS of dark energy with $w_0 = -1$ in Fig.1, where $b = 0.3, 0.6, 1$, respectively. In this figure, we can see w varies with redshift z and b . With the increase of b , the decline of w becomes fast.

Substituting the function Eq. (24) into Eq. (21) and Eq. (22), we obtain that

$$\Omega_2 = \frac{\Omega_{20}}{\Omega_{20} + (1 - \Omega_{20})(1 + b \ln(1 + z))^{3w_0/b}}, \quad (25)$$

$$\Omega_x = 1 - \Omega_2. \quad (26)$$

where, Ω_{20} is the current value at $z = 0$. From Eq. (25) and Eq. (26), we can obtain the evolution of density parameter Ω_2 and Ω_x . Now Ω_x is described in Fig.2 and it is shown that Ω_x increases with decrease of redshift z and the larger b is, the faster Ω_x increases.

We plot the evolutions of Ω_2 and Ω_x with redshift z in Fig. 3 where $b = 0.3, 1$. It is found that two lines intersect at one point. This point is equilibrium of Ω_2 and Ω_x and it varies with b , i.e. the corresponding redshift at the point increases with the growing of b .

Also, we obtain the deceleration parameter q_2 form Eq. (14)

$$q_2 = \frac{1}{2} \left\{ \frac{3w_0[1 + b \ln(1 + z)]^{(\frac{3w_0}{b}-1)}}{\frac{\Omega_{20}}{1-\Omega_{20}} + [1 + b \ln(1 + z)]^{\frac{3w_0}{b}}} + 1 \right\}. \quad (27)$$

The evolution of deceleration parameter q_2 with redshift z is plotted in Fig. 4. The larger b is, the faster the attenuation of q is. The transition from decelerated expansion to accelerated expansion can be seen easily and the redshift z of the point at $q = 0$ becomes smaller with the increase of b .

4. Conclusions

The exact global solutions of brane universes are studied. The solutions contain two arbitrary functions μ and ν . In this paper, we study the Friedmann equation modified on the brane, and the term with ν in the Friedmann equation can drive our universe to accelerate. We assume this term with ν as the density of dark energy on $y \neq 0$ brane. If different ν is given, different models of dark energy can be obtained. We suppose only matter on the brane i.e. $p_2 = 0$. Using Wetterich's parametrization equation of state of dark energy and the relation $A_2 = A_{20}/(1 + z)$, we obtain the relation of ν with redshift z . Thus, if the current values of the two density parameters Ω_{20} , Ω_{x0} , w_0 and the bending parameter b are known, the arbitrary ν could be determined uniquely, and then $\mu(z)$ could be determined too. In this way, the whole $5D$ solutions could be reconstructed. And, in principle the $5D$ solutions could provide with us a global brane cosmological model to simulate our real universe. We have also studied the evolutions of matter density Ω_2 , dark energy density Ω_x and deceleration parameter q with redshift z and different b . These cosmic parameters depend on the bending parameter b . Therefore, we expect accurate observational constraints on current cosmic parameters and bending parameter b in order to determine the evolution of $5D$ global brane universe.

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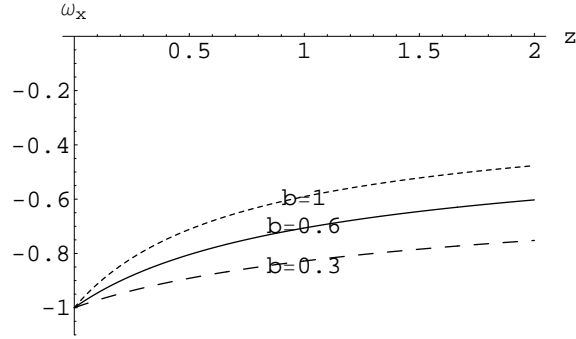


Fig. 1. w_x of the dark energy as a function of the redshift z with its current value $w_0 = -1$ and the bending parameter $b = 1, 0.6, 0.3$ respectively.

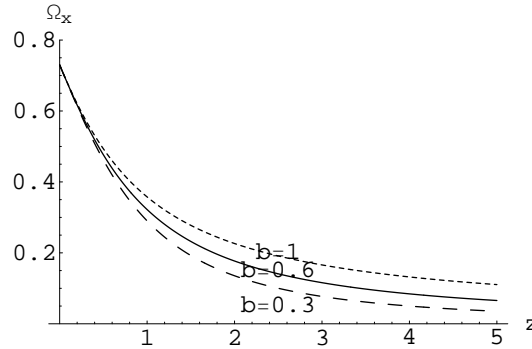


Fig. 2. Evolution of dark energy Ω_x vs. redshift z with $w_0 = -1$, $\Omega_{20} = 0.27$, $\Omega_{x0} = 0.73$ and the bending parameter $b = 1, 0.6, 0.3$ respectively.

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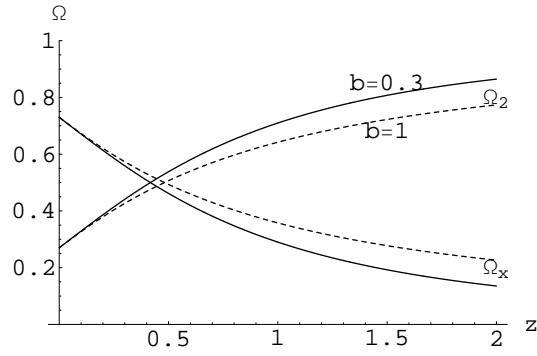


Fig. 3. Evolution of Ω_x and Ω_2 vs. redshift z with $w_0 = -1$, $\Omega_{20} = 0.27$, $\Omega_{x0} = 0.73$ and the bending parameter $b = 1, 0.3$.

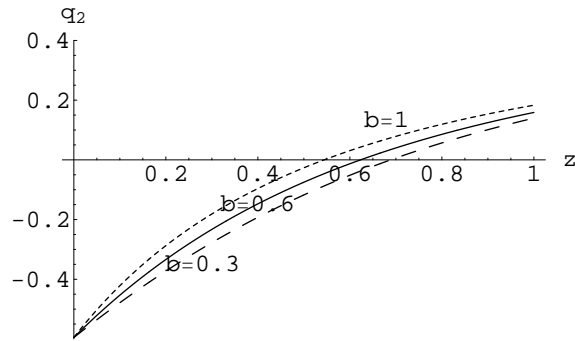


Fig. 4. Evolution of the deceleration parameter q_2 vs. redshift z with $w_0 = -1$, $\Omega_{20} = 0.27$, and the bending parameter $b = 1, 0.6, 0.3$ respectively.